Comment to "Mechanism for Designing Metamaterials with a High Index of Refraction"

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In a recent Letter [1] Shen et al. presented a mechanism for designing metamaterials with a high index of refraction. The proposed metamaterial consists of a metal with tiny slits in it (Fig. 1). This system is suggested to be equivalent to a dielectric with an effective refractive index of $n_{eff} = d/a$ and effective thickness $L_{eff} = L/n_{eff}$.

In an attempt to prove these ideas experimentally, one of us (AP) carried out transmittance and reflectance experiment of the metallic metamaterial with slits [2]. In these experiments the predictions of the theory of Ref. [1] concerning the amplitudes of the transmittance and reflectance have been confirmed. However, the discrepancy of the transmittance phase shift with the predictions of the model has been observed: the experimental phase had to be corrected by a value $\phi_{corr} = 2\pi(L - L_{eff})/\lambda$, with λ being the wavelength of the radiation.

In this Comment we suggest another explanation for the observed effects. Namely, the metamaterial possesses an effective dielectric permittivity equal to $\varepsilon_{eff} = d/a$ and an effective magnetic permeability $\mu_{eff} = a/d$. Therefore, this metamaterial is in fact a low-impedance material with $z_{eff}/z_0 = a/d$ and the refractive index equal to unity, $n_{eff} = 1$. Here $z_0 = \sqrt{\mu_0/\varepsilon_0}$ is the wave impedance of the free space.

In order to demonstrate this idea we consider a quasi-static approximation with the electric fields (E) perpendicular to the slits and magnetic fields (H) parallel to the slits (Fig. 1). In the following we have dropped the corresponding indices and use the field averaging over the unit cell [3]. For electric fields within the metamaterial we utilize the boundary condition in which the normal component of the electric displacement $D = \varepsilon_0\varepsilon E$ is continuous at the slit boundary. Taking into account that: i) D remains constant within the unit cell, ii) in the air slit $D = \varepsilon_0 E$, and iii) within the metal $E = 0$, we get for the average electric field $\varepsilon_0\langle E\rangle/D = \varepsilon_{eff}^{-1} = a/d$. The effective magnetic permeability of the metamaterial may be calculated in a similar way using the continuity of the tangential component of the magnetic field H. In that case the flux density $B = \mu_0\mu H$ and not the field H is zero within the metal. We get $\langle B\rangle/\mu_0 H = \mu_{eff} = a/d$. Similar expressions for permittivity and permeability have been obtained in Ref. [4].

The ability of the present model to describe the experimental data is demonstrated in Fig. 1, which compares the phase shift ϕ as obtained within both models. The most important point here is that the sample thickness should be known during the measurements and, therefore, the *model assumptions influence already the experimental values*. Because the sample thickness is different in both models, the data vary by $\Delta(\phi\lambda/2\pi) = L - L_{eff}$.

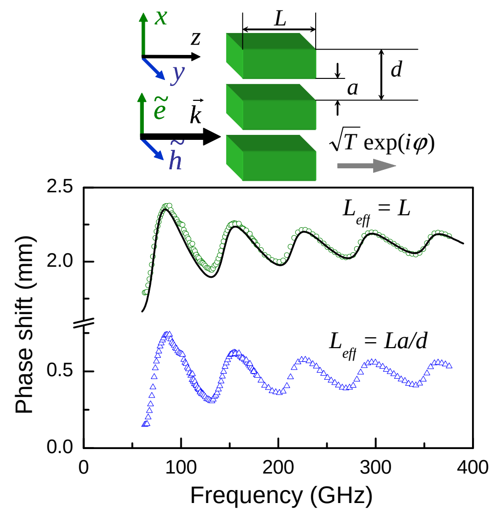

FIG. 1: Experimental phase shift $\phi\lambda/2\pi$ of the metamaterial shown in the inset. Symbols represent the experimental data from Ref. [2]. Green circles - sample thickness is taken as L (this Comment); blue triangles - sample thickness is taken as La/d (Ref. [1]). Both models give the same theoretical values for the phase shift (black solid line), but influences the experiment in different ways. The parameters are: $L = 2.01$ mm, $d/a = 6$ (the fits utilizes $(d/a)_{eff} = 5.5$). Inset shows the geometry of the sample and experiment. The sample is supposed to be infinite in the y-direction.

Finally we note that the amplitudes of the reflectance and transmittance calculated in Ref. [1] remain correct, because the relevant reflection coefficient at the metamaterial surface is equal to $r = (1 - z)/(1 + z) = (d-a)/(d+a)$, i.e. the same expression as used in Ref. [1]. The positions of the Fabry-Pérot resonances remains the same as well, because the optical thickness of the metamaterial $L_{opt} = nL_{eff}$ is the same in both models.

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[3] D. Smith and J. B. Pendry, J. Opt. Soc. Am. B **23**, 391 (2006).
[4] F. J. Garcia-Vidal, L. Martín-Moreno, and J. B. Pendry, J. Opt. A **7**, S97 (2005).